8. *Rutherford's experiment, which established the nuclear model of the atom, used a beam of :*

(A) *β-particles, which impinged on a metal foil and got absorbed*
(B) *γ-rays, which impinged on a metal foil and ejected electrons*
(C) *Helium atoms, which impinged on a metal foil and got scattered*
(D) *Helium nuclei, which impinged on a metal foil and got scattered.*

9. *Of the following, radiation with maximum wavelength is :*

(A) *UV* **(B)** *Radio wave* **(C)** *X-rays* **(D)** *IR*

10. *When a certain metal was irradiated with light of frequency* 3.2×10^{16} *Hz, photoelectrons emitted had twice the kinetic energy as did photoelectrons emitted when the same metal was irradiated with light of frequency* 2.0×10^{16} *Hz. Hence threshold frequency is :*

(A) $0.8 \times 10^{15}\ Hz$ **(B)** $8.0 \times 10^{15}\ Hz$ **(C)** $0.8 \times 10^{14}\ Hz$ **(D)** $6.4 \times 10^{16}\ Hz$

11. *How may photons are emitted per second by a 10 mW laser source operating at 626 nm ?*

(A) 1.6×10^{16} **(B)** 1.6×10^{18} **(C)** 3.2×10^{16} **(D)** *None of these*

12. *If the frequency of light in a photoelectric experiment is doubled, the stopping potential will :*

(A) *be doubled* **(B)** *be halved*
(C) *become more than double* **(D)** *become less than double*

ATOMIC SPECTRA OF HYDROGEN AND BOHR'S MODEL — SECTION - 3

It is observed that the atoms of hydrogen in gas discharge tube emit radiations whose spectrum shows line characteristics (line spectra). The line spectra of hydrogen lies in three regions of Electromagnetic Spectrum: *Infra-red*, *Visible and UV* region. In all there are five sets of discrete lines.

The set of lines in the *Visible* region are known as *Balmer Series*, those in *Ultra-Violet* as *Lyman series* and there are three sets of lines in *Infra-red* region : *Paschen*, *Brackett* and *Pfund series*. Balmer and Rydberg gave an empirical relation to define the wavelength of the lines in each series in terms of a parameter called as *Wave Number* denoted by $\bar{\nu}$. The wave number is defined as reciprocal of the wavelength i.e., $\bar{\nu} = \frac{1}{\lambda}$

$$\bar{\nu} = RZ^2 \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

where *n* and *m* are whole numbers; λ : wavelength of spectral line ; $\bar{\nu}$: wave number of spectral line
R : Rydberg constant. The values of *n* and *m* for different spectral lines for each series are listed below.

Region	Spectral line	*n*	*m*
UV	*Lyman Series*	1	2, 3, 4, . . .
Visible	*Balmer Series*	2	3, 4, 5, . . .
Infra-red	*Paschen Series*	3	4, 5, 6, . . .
Infra-red	*Brackett Series*	4	5, 6, ...
Infra-red	*Pfund Series*	5	6, 7, ...
	Humphry Series	6	8, 7, 8 . . .

In Hydrogen atom spectra :

- Intermediate frequencies were emitted i.e. only specific spectral lines are there in the spectrum (Planck's quantum theory).
- Lines observed were characteristic of Hydrogen atom only.

These observations led Bohr to conclude that electrons in an atom are not randomly distributed, but were arranged in definite energy states. The energy of each state (or level) was fixed or quantised (from characteristic nature of H-atom spectra). The complete theory developed by him is organised in his postulates.

Bohr's Postulates

Bohr's theory was based on the application of Planck's Quantum theory on the atomic spectra of Hydrogen atom. The fundamental postulates of his theory are discussed below :

- The electron in an atom has only certain definite stationary states of motion allowed to it, called as *energy levels*. Each energy level has a definite energy associated with it. In each of these energy levels, electrons move in circular orbit around the positive nucleus. The necessary centripetal force is provided by the electrostatic attraction of the protons in the nucleus. As one moves away from the nucleus, the energy of the states increases.
- These states of allowed electronic motion are those in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$ or one can say that the angular momentum of an electron is quantised.

$\Rightarrow$ Angular momentum $= mvr = n\left(\frac{h}{2\pi}\right)$ Angular momentum = moment of Inertia × angular velocity

$$= mr^2 \times \frac{v}{r} = mvr$$

where *m* is the mass of the electron, v is the velocity of the electron, *r* is the radius of the orbit, h is Planck's constant and *n* is a positive integer.

- When an atom is in one of these states, it does not radiate any energy but whenever there is a transition from one state to other, energy is emitted or absorbed depending upon the nature of transition.

When an electron jumps from higher energy state to the lower energy state, it emits radiations in form of photons or quanta. However, when an electron moves from lower energy state to a higher state, energy is absorbed, again in form of photons.

The energy of a photon emitted or absorbed is given by using Planck's relation ($E = h\nu$). If E_1 be the energy of any lower energy state and E_2 be the energy of any higher energy state, then the energy of the photon (emitted or absorbed) is given as ΔE (i.e., the difference in the energies of two states) : $\Delta E = E_2 - E_1 = h\nu = h\frac{c}{\lambda}$

where *h* : Planck's constant and ν : frequency of radiation emitted or absorbed.

Additional Information :

Coulomb's Law of Electrostatic force of attraction or repulsion (*F*) between two charges q_1 and q_2 separated by a distance '*r*' is given by : Force $(F) = \frac{K|q_1||q_2|}{r^2}$ where $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$

Note that charge on any particle can only be an integral multiple of charge on an electron (e).

Electrostatic Potential energy (E.P.E.) of a system of two charges separated by a distance 'r' is given by :

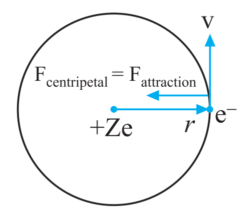
$$\text{E.P.E.} = \frac{Kq_1q_2}{r}$$

Note : E.P.E. is +ve when charges are like and –ve when charges are opposite.

Electrostatic force (F) is repulsive when both q_1 and q_2 are of same sign (i.e. either both are positive or both are negative) and is attractive when q_1 and q_2 are of different signs.

Bohr Model :

Consider a species of atomic number (Z) containing single electron revolving around its nucleus at a distance of 'r' as shown in the figure.



Note : Atomic number ≡ Number of protons on the nucleus = Z

⇒ Charge on the nucleus = + Ze [As charge on each proton is +e and neutrons don't have any charge]

Electrostatic force of attraction (F) between the nucleus of charge + Ze and electron (–e) is given by :

$$F = \frac{K|Ze||-e|}{r^2} = \frac{KZe^2}{r^2} \quad \text{. . . (i)}$$

The centrifugal forces acting on the electron is $\frac{m_e v^2}{r}$ . . . (ii)

[Assuming uniform circular motion]

This centrifugal force must be provided by the electrostatic force of attraction (F).

⇒ From (i) and (ii), we have :

$$\frac{KZe^2}{r^2} = \frac{m_e v^2}{r} \quad \text{. . . (iii)}$$

Also, according to Bohr's postulate of quantization of angular momentum, we have :

Angular momentum of electron about the nucleus = $m_e vr = \frac{nh}{2\pi}$ . . . (iv)

where 'n' is a positive integer

($n = 1, 2, 3, \cdots \infty$)

Solve (iii) and (iv) to get :

$$v = \frac{2\pi KZe^2}{nh} \quad \text{and} \quad r = \frac{n^2h^2}{4\pi^2 K m_e e^2 Z}$$

Put $K = 9 \times 10^9$ Nm^{2}C^{-2}, $e = 1.6 \times 10^{-19}$ C and $h = 6.626 \times 10^{-34}$ Js in the above expressions to get :

Velocity of an electron in nth Bohr orbit $\equiv v_n = 2.18 \times 10^6 \frac{Z}{n}$ ms^{-1}

and Radius of the nth Bohr orbit $\equiv r_n = 0.529 \times 10^{-10} \frac{n^2}{Z}$ m $\equiv 0.529 \frac{n^2}{Z}$ Å $\equiv 52.9 \frac{n^2}{Z}$ pm $\left[1\,\text{pm} = 10^{-12}\,\text{m}\right]$

Now, the Total Energy of the electron moving in nth orbit $\equiv$ K.E.$_n$ + E.P.E.$_n$

$$\text{T.E.}_n = \frac{1}{2} m v_n^2 + \frac{K(Ze)(-e)}{r} \qquad \left[\because \ \text{E.P.E.} \equiv \frac{Kq_1q_2}{r}\right]$$

$$\Rightarrow \quad T.E_n = \frac{1}{2}\left(\frac{KZe^2}{r_n}\right) + \frac{K(Ze)(-e)}{r_n} \qquad \text{[Using (iii)]}$$

$$\Rightarrow \quad E_n \equiv T.E._n = \frac{-KZe^2}{2r_n}$$

It can be shown from the above expressions that :

$$K.E._n = \frac{1}{2}\frac{KZe^2}{r_n}, \quad P.E._n = \frac{-KZe^2}{r_n} \quad \text{and} \quad E_n = \frac{-KZe^2}{2r_n}$$

or $\quad K.E._n = -E_n$ and $E.P.E._n = 2E_n$

Using the value of r_n in the expression of E_n, we get :

$$E_n = \frac{-2\pi^2K^2m_ee^4Z^2}{n^2h^2}$$

$$E_n = -2.18\times10^{-18}\frac{Z^2}{n^2}\,J/atom = -13.6\frac{Z^2}{n^2}\,eV/atom \qquad [\because\ 1eV = 1.6\times10^{-19}J]$$

$$= -2.18\times10^{-18}\frac{Z^2}{n^2}\times6.02\times10^{23}\ J/mole \quad = -1312\frac{Z^2}{n^2}\ kJ/mole$$

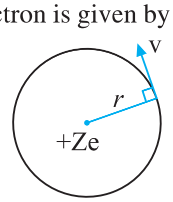
Note : ➤ Bohr's Model is applicable only to one-electron atoms like : He^+, Li^{2+}, Be^{3+} apart from H-atom.

Illustration - 5 *Determine the frequency of revolution of the electron in 2nd Bohr's orbit in hydrogen atom.*

SOLUTION :

The frequency of revolution of electron is given by :

$$\text{Frequency} = \frac{1}{\text{time period}}$$



Time period

$$= \frac{\text{Total distance covered in 1 revolution}}{\text{velocity}} = \frac{2\pi r}{v}$$

$$\text{Hence frequency} = \frac{v}{2\pi r}$$

Calculate velocity (v_2) and radius (r_2) for electron in 2nd Bohr orbit in H-atom (Z = 1)

Z = 1 for H-atom.

Using $\quad r_n = 0.529\frac{n^2}{Z}\,\text{Å}$

$$r_2 = 0.529\times10^{-10}\frac{(2)^2}{1}\,m = 1.12\times10^{-10}\,m$$

$$v_n = 2.18\times10^6\,(1/n)\ m/s$$

$$v_2 = 2.18\times10^6\,(1/2) = 1.09\times10^6\ m/s$$

$$\text{Hence frequency} = \frac{v_2}{2\pi r_2} = \frac{1.09\times10^6}{2(\pi)(2.12\times10^{-10})}$$

$$v = 8.18\times10^{14}\ Hz.$$

Note : Frequency of revolution (f) = 1/T $\qquad$ where $T = \frac{2\pi r}{v} \propto \frac{n^3}{Z^2}\left[\because\ r \propto \frac{n^2}{Z} \text{ and } v \propto \frac{Z}{n}\right]$

$$\Rightarrow \quad f \propto \frac{Z^2}{n^3}$$

What does the negative electron energy (E_n) means ?

The energy of the electron in a hydrogen atom has a negative sign for all possible orbits. What do this negative sign convey ? This negative sign means that the energy of the electron in the atom is lower than the energy of a free electron at rest. An electron in an atom is because of attractive force due to protons in the nucleus. A free electron at rest is an electron that is infinitely far away from the nucleus and is assigned the energy value of zero. Mathematically, this corresponds to setting n equal to infinity in the equation so that $E_\infty = 0$. As electron gets closer to the nucleus, E_n becomes larger in absolute value and more and more negative. The most negative energy value is given by $n = 1$ which corresponds to the most stable orbit.

When an electron jumps from an outer orbit (higher energy) $\boldsymbol{n_2}$ to an inner orbit (lower energy) $\boldsymbol{n_1}$, then the energy emitted in form of radiation is given by :

$$\Delta E = E_{n_2} - E_{n_1} = \frac{2\pi^2 K^2 me^4 Z^2}{h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \Rightarrow \quad \Delta E = 2.18 \times 10^{-18} \times Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Also, $$\Delta E = 13.6 \times Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{ eV/atom}$$

As we know that : $E = h\nu \quad ; \quad \bar{\nu} = \frac{1}{\lambda}$ $\quad \Rightarrow \quad$ $$\bar{\nu} = \frac{\Delta E}{hc} = \frac{2\pi^2 K^2 me^4 Z^2}{ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The above equation can be represented as : $\bar{\nu} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ where $R = \frac{2\pi^2 K^2 me^4}{ch^3}$

R is known as Rydberg constant. Its value to be used is = 109677 cm^{-1} = 10967700 m^{-1}

Note : (i) The value of $\frac{1}{R} \approx 911.5 \text{ Å}$ is sometimes useful.

(ii) This relation exactly matches with the empirical relation given by **Balmer** and **Rydberg** to account for the spectral lines in H-atom spectra. In fact the value of Rydberg constant in the empirical relation is approximately the same as calculated from the above relation (**Bohr's Theory**). This was the main success of Bohr's Theory i.e. to account for the experimental observations by postulating a theory.

(iii) The maximum number of lines that can be emitted when an electron in an excited state $n = n_2$ de-excites to a state $n = n_1$ ($n_2 > n_1$) is given by : $\frac{(n_2 - n_1 + 1)(n_2 - n_1)}{2}$

Illustration - 6 *Determine the maximum number of lines that can be emitted when an electron in H atom in n = 6 state drops to the ground sate. Also find the transitions corresponding to the lines emitted.*

SOLUTION :

The maximum number of lines can be calculated by using the above formula with $n_2 = 6$ and $n_1 = 1$ are 15.

The distinct transitions corresponding to these lines are:

$6 \to 1$

$6 \to 2, \quad 2 \to 1$

$6 \to 3, \quad 3 \to 2, \quad 3 \to 1$

$6 \to 4, \quad 4 \to 3, \quad 4 \to 2, \quad 4 \to 1$

$6 \to 5, \quad 5 \to 4, \quad 5 \to 3, \quad 5 \to 2, \quad 5 \to 1$

Note : Each line (in emission spectra) corresponds to a particular photon emitted. The photon with shortest wavelength is corresponding to the largest energy difference ($6 \to 1$) and with longest wave length is corresponding to minimum energy difference ($6 \to 5$).